

LAWS OF EXPONENTS

In the expression 5^2 , 5 is the **base** and 2 is the **exponent**. For x^a , x is the base and a is the exponent. 5^2 means $5 \cdot 5$. 5^3 means $5 \cdot 5 \cdot 5$, so you can write $\frac{5^5}{5^2}$ (which means $5^5 \div 5^2$) or you can write it like this: $\frac{5 \cdot 5 \cdot 5 \cdot 5 \cdot 5}{5 \cdot 5}$.

You can use the Giant **1** to find the numbers in common. There are two Giant **1**s, namely, $\frac{5}{5}$ twice so $\frac{5 \cdot 5 \cdot 5 \cdot 5 \cdot 5}{5 \cdot 5} = 5^3$ or 125. Writing 5^3 is usually sufficient.

When there is a variable, it is treated the same way. $\frac{x^7}{x^3}$

means $\frac{x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x}{x \cdot x \cdot x}$. The Giant **1** here is $\frac{x}{x}$ (three of them). The answer is x^4 .

$5^2 \cdot 5^3$ means $(5 \cdot 5)(5 \cdot 5 \cdot 5)$ which is 5^5 . $(5^2)^3$ means $(5^2)(5^2)(5^2)$ or $(5 \cdot 5)(5 \cdot 5)(5 \cdot 5)$ which is 5^6 .

When the problems have variables such as $x^4 \cdot x^5$, you only need to add the exponents. The answer is x^9 . If the problem is $(x^4)^5$ (x^4 to the fifth power) it means $x^4 \cdot x^4 \cdot x^4 \cdot x^4 \cdot x^4$. The answer is x^{20} . You multiply exponents in this case.

If the problem is $\frac{x^{10}}{x^4}$, you subtract the bottom exponent from the top exponent ($10 - 4$).

The answer is x^6 . You can also have problems like $\frac{x^{10}}{x^{-4}}$. You still subtract, $10 - (-4)$ is 14, and the answer is x^{14} .

You need to be sure the bases are the same to use these laws. $x^5 \cdot y^6$ cannot be further simplified.

In general the laws of exponents are:

$$x^a \cdot x^b = x^{(a+b)}$$

$$(x^a)^b = x^{ab}$$

$$\frac{x^a}{x^b} = x^{(a-b)}$$

$$x^0 = 1$$

$$(x^a y^b)^c = x^{acy} y^{bc}$$

These rules hold if $x \neq 0$ and $y \neq 0$.

For additional information, see Year 2, Chapter 8, problem GS-26 and Chapter 10, problem MG-35.

Examples

a) $x^8 \cdot x^7 = x^{15}$ b) $\frac{x^{19}}{x^{13}} = x^6$ c) $(z^8)^3 = z^{24}$
d) $(x^2y^3)^4 = x^8y^{12}$ e) $\frac{x^4}{x^{-3}} = x^7$ f) $(2x^2y^3)^2 = 4x^4y^6$
g) $(3x^2y^{-2})^3 = 27x^6y^{-6}$ or $\frac{27x^6}{y^6}$ h) $\frac{x^8y^5z^2}{x^3y^6z^{-2}} = \frac{x^5z^4}{y}$ or $x^5y^{-1}z^4$

Problems

Simplify each expression.

1. $5^2 \cdot 5^4$ 2. $x^3 \cdot x^4$ 3. $\frac{5^{16}}{5^{14}}$ 4. $\frac{x^{10}}{x^6}$ 5. $(5^3)^3$
6. $(x^4)^3$ 7. $(4x^2y^3)^4$ 8. $\frac{5^2}{5^{-3}}$ 9. $5^5 \cdot 5^{-2}$ 10. $(y^2)^{-3}$
11. $(4a^2b^{-2})^3$ 12. $\frac{x^5y^4z^2}{x^4y^3z^2}$ 13. $\frac{x^6y^2z^3}{x^{-2}y^3z^{-1}}$ 14. $4x^2 \cdot 2x^3$

Answers

1. 5^6 2. x^7 3. 5^2 4. x^4 5. 5^9
6. x^{12} 7. $256x^8y^{12}$ 8. 5^5 9. 5^3 10. y^{-6} or $\frac{1}{y^6}$
11. $64a^6b^{-6}$ or $\frac{64a^6}{b^6}$ 12. xy 13. $\frac{x^8z^4}{y}$ or $x^8y^{-1}z^4$ 14. $8x^5$